OPEN-BOOK EXERCISE

for

ECONOMICS VISITING STUDENT APPLICANTS

To be completed by all candidates applying to take courses in Economics

Lady Margaret Hall
University of Oxford

Please carefully follow the directions about which questions to attempt, according to the prerequisites for your requested course choices.

We recommend that you spend no more than one hour attempting each question.

You are welcome to make use of your course notes and textbooks, but you should not seek assistance from any other person. You are welcome to use a calculator.

This is not an examination: the main purpose of this exercise is to assess whether you meet the prerequisites for the Oxford economics courses that you have requested, at the point of application. We may be able to offer suitable alternatives if you do not, and will take into account the courses that you plan to take between application and admission. Please attempt each question to the best of your ability, given the courses that you have studied thus far. If you are not familiar with a concept or topic, please write a note to that effect and we will take that into account. You are welcome to attempt questions on topics that you have not previously studied, but there is no need to spend time learning new material.
Please fill in the table below, then turn to the indicated parts of this document and follow the instructions. This table should be submitted with your application.

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<th>Oxford Prerequisite</th>
<th>Please tick if required for any of your requested courses</th>
<th>If required for any of your requested courses:</th>
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<td></td>
<td>(i) please tick the appropriate option</td>
<td>(ii) please then complete the part indicated</td>
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<tr>
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<td>I have already completed an equivalent</td>
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<td>(H) Introduction to Game Theory</td>
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Part A (Calculus for Economics)

US equivalent: Any calculus course that covers basic integration, partial differentiation and the implicit function theorem (informally)

A1. Please follow these instructions:

- If you have already completed an equivalent course:
  o Please state the name of the course, the date of completion and the grade attained.
  o Please append the course syllabus or synopsis.
  o Please then complete question A2 below.
- If you plan to complete an equivalent before arrival in Oxford:
  o Please state the name of the course and the expected date of completion.
  o Please append the course syllabus or synopsis.
  o You do not need to complete question A2 below.
- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete question A2 below.

A2. Please attempt each part of this question.

(i) Solve the simultaneous equations $y = x^2 + 1$ and $2y - 3x = 4$.
(ii) Find all roots of the equation $x^3 - 2x^2 - 9x + 18 = 0$.
(iii) Simplify $\frac{1}{2} \ln x^4 + 3 \ln (2y)$.
(iv) Evaluate $2 + 1 + 0.5 + 0.25 + 0.125 + \ldots$.
(v) Find all stationary points of the function $f(x, y) = x^3 - y^3 - 3x + 12y$, and classify them.
(vi) Find the maximum and the minimum of the function $f(x, y) = 2x + y^2$ subject to the constraint $x + y = 4$, where $x$ and $y$ are both non-negative real numbers.
Part B (Introductory Probability & Statistics)

US equivalent: A one-semester course, possibly called "Statistics for Economics" or "Statistics for Economics and Business" that satisfies the home university's prerequisite for "Econometrics"

B1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete question B2 below.
- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete question B2 below.
- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete question B2 below.

B2. Please attempt each part of this question.

(i) In 2010, 24% of PPE finalists in Oxford achieved First class honours, 70% achieved a 2:1 and 6% achieved a 2:2 (no-one failed or achieved a 3rd class degree that year). PPEists are allowed to drop one of the three subjects for Finals, and 40% of that cohort had dropped Economics. Only 15% of those students who had dropped Economics achieved a First. What is the probability that a PPE student who achieved a First took Economics? (Note: undergraduate degrees in the UK are classified as First, 2:1, 2:2 or Third class honours. The PPE degree combines Philosophy, Politics and Economics.)

(ii) Define and explain what it means for an estimator to be “consistent” and “efficient”, giving examples and counter-examples.

(iii) Show that the residual $e_i$ in the identity $Y_i = E[Y_i | X_i] + e_i$ is mean independent of $X_i$.

(iv) Using the potential outcomes framework, explain what is meant by selection bias.
Part C (Micro Theory I)

US equivalent: Intermediate Micro (with calculus) OR Intermediate Micro PLUS Calculus for Economics

Note: At some universities (but very few) the 'Principles of Microeconomics' may reach the required level. Students should submit a detailed syllabus for review if they wish their Principles course, or another course, to be considered.

C1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions C2 and C3 below.

- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions C2 and C3 below.

- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions C2 and C3 below.

C2. Please attempt each part of this question.

(i) If the interest rate is 4%, find the present value of a perpetuity that pays £200 every year.

(ii) Find all roots of the equation $x^3 - 2x^2 - 9x + 18 = 0$.

(iii) Find and classify all stationary points of the function $f(x, y) = x^3 - y^3 - 3x + 12y$.

(iv) Find the maximum and the minimum of the function $f(x, y) = 2x + y^2$ subject to the constraint $x + y = 4$, where $x$ and $y$ are both non-negative real numbers.

(v) A monopolist faces demand $q(p)$. Find the elasticity of demand at the revenue-maximising quantity.

(vi) Why do perfectly competitive firms produce when they don’t make any profit in the long run?
C3. Arthur lives in a world in which there are just two goods: nutmegs and pears. In his garden there is a tree which yields eight nutmegs and one pear every day. He has no income other than from his nutmegs and pears. Arthur’s preferences over nutmegs and pears may be represented by the utility function \( u(n, p) = 2 \ln n + 3 \ln p \), where \( n \) is the number of nutmegs that he consumes and \( p \) is the number of pears.

(i) Show that Arthur’s preferences may, alternatively, be represented by a utility function of the form \( v(n, p) = n^\alpha p^\beta \) and explain why this is the case.

(ii) People in Arthur’s world are prepared to trade one pear for two nutmegs. Describe the relationship between the prices of pears and nutmegs and draw a carefully labelled graph of Arthur’s budget constraint.

(iii) Arthur maximises his utility, subject to his budget constraint. Show that his gross demands are 4 nutmegs and 3 pears. Mark the gross demands on your diagram and sketch in one or two of his indifference curves. What are his net demands?

(iv) There is a shortage of nutmegs in Arthur’s world, so the relative price of nutmegs increases. (Arthur’s tree still produces the same yield every day.) Illustrate on your diagram what happens to his budget constraint. Will he be better off or worse off after the price change? What can you say about how his gross demands will change?

(v) Consider the effect of the change in the price of nutmegs on Arthur’s demand for nutmegs. This may be decomposed into a substitution effect, an ordinary income effect and an endowment income effect. Explain what is meant by these terms and draw a diagram to illustrate this decomposition. (Please draw a new graph for this part of the question.)
Part D (Macro Theory I)

US equivalent: Intermediate Macro (with calculus) OR Intermediate Macro PLUS Calculus for Economics

Note: At some universities (but very few) the 'Principles of Macroeconomics' may reach the required level. Students should submit a detailed syllabus for review if they wish their Principles course, or another course, to be considered.

D1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions D2 and D3 below.

- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions D2 and D3 below.

- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions D2 and D3 below.

D2. Suppose that consumers live for 2 periods (the present and future). Each consumer has income $y_1$ in the present and $y_2$ in the future, can borrow and save at the real interest rate $r$ and has well-behaved preferences over current and future consumption, $c_1$ and $c_2$.

(i) Write down the consumer's budget constraint and draw a diagram to illustrate the optimal choice of consumption over the two periods.

(ii) Explain carefully why, according to this model, changes in the interest rate may have little effect on saving.

(iii) Discuss how a temporary increase in income, in the present period only, has different effects from an increase in the same size that is expected to be permanent. What would be the effect of a temporary rise in income if there were many periods in the model? What are the implications of your findings for the marginal propensity to consume?

(iv) Suppose the government levies a tax $T$ on each consumer in the present, invests the proceeds in bonds paying interest at $r$ and returns the amount $T$ plus interest, to the consumer in the future. How will this affect $c_1$ and $c_2$? Would the answer be different if consumers faced borrowing constraints?
(v) Compare the policy implications of this model of consumption with those of the Keynesian consumption function.

**D3.** A decline in foreign demand for U.S. goods: Suppose that the European and Japanese economies succumb to a recession and reduce their demand for U.S. goods for several years. Using the AS/AD framework, explain the macroeconomic consequences of this shock, both immediately and over time.
Part E (Micro Theory II)

US equivalent: Advanced Microeconomics

Note: At some universities (but very few) the 'Intermediate Microeconomics' or 'Microeconomic Theory' course may reach the required level. Students should submit a detailed syllabus for review if they wish their Intermediate course, or another course, to be considered.

E1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions E2 and E3 below.

- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions E2 and E3 below.

- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions E2 and E3 below.

E2.

A village has \( n \) residents, who each obtain utility from private goods, and a public good, flood defences. Resident \( i \) has utility function \( U_i = y_i + aQ - \frac{Q^2}{100} \) where \( Q \) is the level of flood defences, and \( y_i \) is individual's \( i \) expenditure on private goods. Flood defence costs \( c \) per unit.

(a) Find the marginal private benefit of flood defences for resident \( i \). Write down and explain the Samuelson condition for optimal provision of the public good.

Hence, the people agree to provide flood defences by voluntary contribution. Resident \( i \) purchases a quantity \( q_i \), taking the contributions of everyone else as given.

(b) Find the reaction function of person 1, as a function of the contributions of others.

(c) How much of the public good will be provided in total, in a symmetric equilibrium in which each person provides the same amount? Why does it not depend on the number of residents?

(d) Discuss your results, illustrating them in a diagram showing the private and social marginal benefits and costs as a function of \( Q \).
Arthur is risk averse, and his income tomorrow depends on which of two possible states occurs; each state is equally likely. His income will be 8 if state 1 occurs, but only 2 if state 2 occurs.

(a) Draw Arthur’s indifference curves in state-contingent income space, and explain how he could be better off if he were able to buy insurance.

Norma is risk neutral. Her income will be 3 if state 1 occurs, and 7 if state 2 occurs. Suppose that they can write contracts of the form “Arthur will give Norma an amount $x$ if and only if state 1 occurs, and Norma will give Arthur an amount $y$ if and only if state 2 occurs.”

(b) What shape are Norma’s indifference curves? From Arthur’s point of view, what is the best contract that Norma would find acceptable, and why?

Now assume that Arthur’s utility as a function of his income $m$ is $\ln m$.

(c) From Norma’s point of view, what is the best contract that Arthur would find acceptable?

(d) Illustrate your results from part (b) and part (c) in an Edgeworth box, and highlight the set of efficient risk-sharing contracts.
Part F (Macro Theory II)

US equivalent: Advanced Macroeconomics

Note: At some universities (but very few) the 'Intermediate Macroeconomics' or 'Macroeconomic Theory' may reach the required level. Students should submit a detailed syllabus for review if they wish their Intermediate course, or another course, to be considered.

F1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions F2 and F3 below.

- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions F2 and F3 below.

- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions F2 and F3 below.

F2.

4. Consider the Phillips curve equation from the IS – PC – MR model

\[ \pi_t = \pi_{t-1} + \alpha(y_t - y_e) + u_t \]

Suppose that in period \( t \) the monetary authority learns of a persistent cost-push shock, i.e. it knows as of period \( t \) that \( u_t = u_{t+1} = 1 \), but then expects the shock to be 0 from \( t + 2 \) onwards.

i. Assuming the economy was in equilibrium at the inflation target before period \( t \), show the position of the economy in the IS – PC – MR diagram in period \( t \).

ii. Assuming that private agents have adaptive expectations whereas the monetary authority is forward-looking, describe the path followed by the economy from period \( t + 1 \) until the inflation target is restored.
F3. Give concise answers to all parts of this question.

(i) Under what conditions will the real interest rate in a small open economy be equal to the world real interest rate? Would your answer be different for nominal interest rates?

(ii) Are government budget deficits always accompanied by current account deficits?

(iii) Suppose that governments set their fiscal plans only through to the end of the current parliamentary term. Does this invalidate Ricardian Equivalence?
Part G (Econometrics)

US equivalent: Econometrics or Applied Econometrics

G1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions G2 and G3 below.
- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions G2 and G3 below.
- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions G2 and G3 below.

G2. Give concise answers to all parts of this question.

(i) Explain how measurement error causes attenuation bias in the linear regression model.

(ii) “Most regression studies rely on the conditional independence assumption in order to argue that the estimated coefficients represent causal effects.” Explain and discuss.

(iii) Consider the following AR(1) time-series model: \( y_t = \alpha + \beta y_{t-1} + \varepsilon_t \). What econometric problems arise if \( \beta = 1 \)? What if \( \beta < 1 \) but close to 1?

(iv) Suppose we had some time-series data and estimated an AR(1) model, obtaining the following:

\[
\begin{align*}
    y_t &= 5.057 + 0.947 y_{t-1} \\
    (2.125) & (0.022)
\end{align*}
\]

where standard errors are reported in parentheses. Can you reject that \( \beta = 1 \) at the 5% significance level?

(v) What problems are caused by structural breaks in time series? How would you test for a structural break?
G3. Answer both parts of this question.

a)

The demand for an agricultural product is described by the function:

$$\ln Q_d = \beta_0 + \beta_1 \ln P_i + u_i$$  (A)

and the supply of an agricultural product is described by the function:

$$\ln Q_s = \gamma_0 + \gamma_1 \ln P_s + \nu_i$$  (B)

(i) Solve for the equilibrium price and quantity in terms of the demand error ($u_i$) and the supply error ($\nu_i$). What do your results tell you about the correlation between $\ln P_i$ and $u_i$?

(ii) If the coefficients of the demand function (A) are estimated by OLS would you expect the estimate of the demand elasticity ($\beta_1$) to be an over or an under-estimate of the true value of $\beta_1$. Explain.

(iii) It is proposed to estimate the demand elasticity by IV using one of the following as an instrument (a) income per capita in the region; (b) a measure of average rainfall in the region. Comment on the appropriateness of each of these variables as an instrument for this problem.

b)

The following table contains the regression output of an investigation relating children's income as adults (aged 35) with the income of their parents (measured when their parents were also aged 35):

<table>
<thead>
<tr>
<th>OLS regression</th>
<th>coefficient</th>
<th>standard error</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(parental income)</td>
<td>0.568</td>
<td>0.005</td>
</tr>
<tr>
<td>living in rural area (=1 if rural; =0 if urban)</td>
<td>-0.091</td>
<td>0.003</td>
</tr>
<tr>
<td>constant</td>
<td>1.94</td>
<td>0.017</td>
</tr>
<tr>
<td>Number of observations</td>
<td>3,056</td>
<td></td>
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</tbody>
</table>

(a) Compute the $R^2$ of this regression and interpret. A researcher claims that the larger the $R^2$ of a regression, the more likely is that the regression has a causal interpretation. Do you agree? Explain.

(b) Interpret the coefficient on the variable log(parental income). Compute and interpret the $p$-value for the hypothesis that the parameter of log(parental income) is zero.

(c) Test at the 1% significance level, the hypothesis that all other things being equal, children living in rural areas have lower income than children living in urban areas. Explain fully the null and alternative hypothesis, test statistic, decision rule and conclusion.
Part G (Introduction to Game Theory)

US equivalent: Any Microeconomic Theory or Game Theory course that covers non-cooperative game theory with discrete and continuous actions, simultaneous, sequential and repeated games and solution concepts including dominance, Nash equilibrium and subgame perfect equilibrium.

H1. Please follow these instructions:

- If you have already completed an equivalent course:
  - Please state the name of the course, the date of completion and the grade attained.
  - Please append the course syllabus or synopsis.
  - Please then complete questions H2 to H4 below.
- If you plan to complete an equivalent before arrival in Oxford:
  - Please state the name of the course and the expected date of completion.
  - Please append the course syllabus or synopsis.
  - You do not need to complete questions H2 to H4 below.
- If you plan to complete this course in Oxford, please ensure that you have included it in your requested course choices. You do not need to complete questions H2 to H4 below.

H2. Give concise answers to all parts of this question.

(a) What does it mean for a strategy to be dominant?
   - If players choose dominant strategies, will they play according to a Nash Equilibrium?
   - And if they play according to a Nash Equilibrium, will they play dominant strategies?

(b) For each of the following two-player one-shot games, discuss whether or not you can predict how the players will play.
   - **Game 1.** Each player decides between Green and Red and they simultaneously announce their choices, which result in the following pay-offs:
     - if they chose the same action then they each receive 5, otherwise the one that chose Green receives 6 and the one that chose Red receives 4.
   - **Game 2.** Each player decides between Blue and Yellow and they simultaneously announce their choices, which result in the following pay-offs:
     - if they both chose Blue then they each receive 3, whereas if they both chose Yellow then they each receive 5; otherwise the one that chose Blue receives 4 and the one that chose Yellow receives 2.

(c) Comment on the usefulness of the Nash Equilibrium concept for predicting the outcome of a game.
H3. Give concise answers to all parts of this question

Consider the following two-player, one-shot, simultaneous-move game:

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<th>Left</th>
<th>Right</th>
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<tbody>
<tr>
<td><em>Up</em></td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td><em>Down</em></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Identify the unique Nash equilibrium, in pure strategies, of this game.

Now consider a variant of the previous game in which the Row player moves first, followed by the Column player.

(b) Represent the game using the extensive form and identify the subgame perfect equilibrium of the game. Is the subgame perfect equilibrium a Nash equilibrium?

(c) What happens if the roles are reversed, and the Row player is the ‘follower’?

d) Consider the two sequential games that you have analysed. Does either have a Nash Equilibrium which is not Subgame Perfect? If so, identify it and explain why it is Nash but not Subgame Perfect.

H4.

The two firms in an industry have identical cost functions \( c(q_k) = 2q_k, \ k = 1, 2 \), and the inverse demand function in the industry is \( p(q) = 14 - q \), where \( q = q_1 + q_2 \).

(a) Verify that the optimal production for a joint-monopoly is \( q^M = 6 \), while the Cournot equilibrium is for each firm to produce \( q^C = 4 \).

Suppose that each firm can simply choose to produce either \( \frac{1}{2}q^M = 3 \) (half the joint-monopoly quantity) or \( q^C = 4 \) (the Cournot quantity).

(b) Write down the one-shot simultaneous-move game corresponding to this simplified set-up and show that each firm has a dominant strategy. Discuss.