LMH
Lady Margaret Hall

OPEN-BOOK EXERCISE

for

ECONOMICS VISITING STUDENT APPLICANTS

To be completed by all candidates applying to take courses in Economics

Lady Margaret Hall
University of Oxford

Please carefully follow the directions about which questions to attempt, according to your course choices.

We recommend that you spend no more than one hour attempting each question.

You are welcome to make use of your course notes and textbooks, but you should not seek assistance from any other person. You are welcome to use a calculator.

This is not an examination: the main purpose of this exercise is to assess whether you meet the prerequisites for the Oxford economics courses that you have requested, at the point of application. We may be able to offer suitable alternatives if you do not, and will take into account the courses that you plan to take between application and admission. Please attempt each question to the best of your ability, given the courses that you have studied thus far. If you are not familiar with a concept or topic, please write a note to that effect and we will take that into account. You are welcome to attempt questions on topics that you have not previously studied, but there is no need to spend time learning new material.

All applicants should attempt Q1.

Q1. Please attempt each part of this question.

(i) Solve the simultaneous equations $y = x^2 + 1$ and $2y - 3x = 4$.

(ii) Find all roots of the equation $x^3 - 2x^2 - 9x + 18 = 0$.

(iii) Simplify $\frac{1}{2}\ln x^4 + 3\ln(2y)$.

Any applicant who wishes to take course #3 but not course #1 should attempt Q2.

Q2. Please attempt each part of this question.

(i) Evaluate $2 + 1 + 0.5 + 0.25 + 0.125 + \ldots$.

(ii) Find all stationary points of the function $f(x, y) = x^3 - y^3 - 3x + 12y$, and classify them.

(iii) Find the maximum and the minimum of the function $f(x, y) = 2x + y^2$ subject to the constraint $x + y = 4$, where x and y are both non-negative real numbers.

Any applicant who wishes to take any course numbered #4 or above, but not course #3, should attempt Q3, Q4 and Q5.

Q3. Please attempt each part of this question.

(i) If the interest rate is 4%, find the present value of a bond that pays £200 every year in perpetuity (a 'perpetuity').

(ii) Find all roots of the equation $x^3 - 2x^2 - 9x + 18 = 0$.

(iii) A monopolist faces demand $q(p)$. Find the elasticity of demand at the revenue-maximising quantity.

(iv) Find and classify all stationary points of the function $f(x, y) = x^3 - y^3 - 3x + 12y$.

(v) Find the maximum and the minimum of the function $f(x, y) = 2x + y^2$ subject to the constraint $x + y = 4$, where x and y are both non-negative real numbers.

Q4. Arthur lives in a world in which there are just two goods: nutmegs and pears. In his garden there is a tree which yields eight nutmegs and one pear every day. He has no income other than from his nutmegs and pears. Arthur's preferences over nutmegs and pears may be represented by the utility function $u(n, p) = 2 \ln n + 3 \ln p$, where n is the number of nutmegs that he consumes and p is the number of pears.

(i) Show that Arthur's preferences may, alternatively, be represented by a utility function of the form $v(n, p) = n^{\alpha} p^{\beta}$ and explain why this is the case.

(ii) People in Arthur's world are prepared to trade one pear for two nutmegs. Describe the relationship between the prices of pears and nutmegs and draw a carefully labelled graph of Arthur's budget constraint.

(iii) Arthur maximises his utility, subject to his budget constraint. Show that his gross demands are 4 nutmegs and 3 pears. Mark the gross demands on your diagram and sketch in one or two of his indifference curves. What are his net demands?

(iv) There is a shortage of nutmegs in Arthur's world, so the relative price of nutmegs increases. (Arthur's tree still produces the same yield every day.) Illustrate on your diagram what happens to his budget constraint. Will he be better off or worse off after the price change? What can you say about how his gross demands will change?

(v) Consider the effect of the change in the price of nutmegs on Arthur's demand for nutmegs. This may be decomposed into a substitution effect, an ordinary income effect and an endowment income effect. Explain what is meant by these terms and draw a diagram to illustrate this decomposition. (Please draw a new graph for this part of the question.)

Q5. Please write an essay of up to two pages in length in response to part (i) OR part (ii) of this question.

(i) How can all the oligopoly models be right, when their predictions are so different?

(ii) Why do perfectly competitive firms produce when they don't make any profit in the long run?

Any applicant who wishes to take any course numbered #5 or above, but not course #4, should attempt Q6 and Q7.

Q6. Suppose that consumers live for 2 periods (the present and future). Each consumer has income y_1 in the present and y_2 in the future, can borrow and save at the real interest rate r and has well-behaved preferences over current and future consumption, c_1 and c_2.

(i) Write down the consumer's budget constraint and draw a diagram to illustrate the optimal choice of consumption over the two periods.

(ii) Explain carefully why, according to this model, changes in the interest rate may have little effect on saving.

(iii) Discuss how a temporary increase in income, in the present period only, has different effects from an increase in the same size that is expected to be permanent. What would be the effect of a temporary rise in income if there were many periods in the model? What are the implications of your findings for the marginal propensity to consume?

(iv) Suppose the government levies a tax T on each consumer in the present, invests the proceeds in bonds paying interest at r and returns the amount T plus interest, to the consumer in the future. How will this affect c_1 and c_2? Would the answer be different if consumers faced borrowing constraints?

(v) Compare the policy implications of this model of consumption with those of the Keynesian consumption function.

Q7. 'When expectations are formed adaptively there is a short-run trade-off between unemployment and inflation, but when expectations are formed rationally there is no short-run trade-off between unemployment and inflation.' Discuss. *(Please write an essay of up to two pages in length in response to this statement.)*

Any applicant who wishes to take course #7, #8, #10, #12, #13 or #14, should attempt Q8.

Q8. Give concise answers to all parts of this question.

(i) Derive Bayes' Theorem from the definition of conditional probability.

(ii) Alice the meteorologist determines that the probability of rain on Saturday is 50%, and the probability of rain on Sunday is also 50%. Bob the presenter sees Alice's forecast and summarises it as follows: "According to Alice we're in for a wet weekend. There's a 100% chance of rain this weekend: 50% on Saturday and 50% on Sunday." Is Bob correct? Why or why not?

(iii) Approximately what is the probability that a standard normal random variable will take on a value between -1 and 1?

(iv) True or false? The following statements are equivalent:

a. Variables A and B are independent

b. The covariance and correlation between variables A and B are zero.

(v) Suppose that $X_1, \ldots, X_n$ are iid draws from a population with mean 2 and variance 9 and define $\bar{X} \equiv \frac{1}{n}\sum_{i=1}^{n} X_i$. Calculate $\mathbb{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$ in terms of n.

(vi) Define and explain what it means for an estimator to be "consistent" and "efficient".

(vii) Using the potential outcomes framework, explain what is meant by selection bias.

Any applicant who wishes to take course #8, #10, #12, #13 or #14, should attempt Q9 and Q10.

Q9. Give concise answers to all parts of this question.

(i) Show that the residual e_i in the identity $Y_i = E[Y_i \mid X_i] + e_i$ is mean independent of X_i.

(ii) Explain how measurement error causes attenuation bias in the linear regression model.

(iii) "Most regression studies rely on the conditional independence assumption in order to argue that the estimated coefficients represent causal effects." Explain and discuss.

(iv) Consider the following AR(1) time-series model: $y_t = \alpha + \beta y_{t-1} + \varepsilon_t$. What econometric problems arise if $\beta = 1$? What if $\beta < 1$ but close to 1?

(v) Suppose we had some time-series data and estimated an AR(1) model, obtaining the following:

$$\underset{}{y_t} = \underset{(2.125)}{5.057} + \underset{(0.022)}{0.947} y_{t-1}$$

where standard errors are reported in parentheses. Can you reject that $\beta = 1$ at the 5% significance level?

(vi) What problems are caused by structural breaks in time series? How would you test for a structural break?

Q10. Answer both parts of this question.

Part A

The demand for an agricultural product is described by the function:

$$\ln Q_i = \beta_0 + \beta_1 . \ln P_i + u_i \qquad \text{(A)}$$

and the supply of an agricultural product is described by the function:

$$\ln Q_i = \gamma_0 + \gamma_1 . \ln P_i + v_i \qquad \text{(B)}$$

(i) Solve for the equilibrium price and quantity in terms of the demand error (u_i) and the supply error (v_i). What do your results tell you about the correlation between $\ln P_i$ and u_i ?

(ii) If the coefficients of the demand function (A) are estimated by OLS would you expect the estimate of the demand elasticity (β_1) to be an over or an under-estimate of the true value of β_1. Explain

(iii) It is proposed to estimate the demand elasticity by IV using one of the following as an instrument (a) income per capita in the region; (b) a measure of average rainfall in the region. Comment on the appropriateness of each of these variables as an instrument for this problem.

Part B

The following table contains the regression output of an investigation relating children's income as adults (aged 35) with the income of their parents (measured when their parents were also aged 35):

OLS regression

Dependent variable: `log(children income)`

	coefficient	standard error
`log(parental income)`	0.568	0.005
`living in rural area (=1 if rural; =0 if urban)`	−0.091	0.003
`constant`	1.94	0.017
Number of observations	3,056	

	Residual	Total
Sum of Squares	13.42	73.01

(a) Compute the R^2 of this regression and interpret. A researcher claims that the larger the R^2 of a regression, the more likely is that the regression has a causal interpretation. Do you agree? Explain.

(b) Interpret the coefficient on the variable `log(parental income)`. Compute and interpret the p-value for the hypothesis that the parameter of `log(parental income)` is zero.

(c) Test at the 1% significance level, the hypothesis that *all other things being equal*, children living in rural areas have lower income than children living in urban areas. Explain fully the null and alternative hypothesis, test statistic, decision rule and conclusion.